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ROBUST FIBER OPTIC TOPOLOGIES

Massachusetts Institute of Technology

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The intent of this program was to investigate how single mode fiber optic technology can support a flexible, redundant and therefore robust (survivable) network. This study focused on the signal processing that should be performed at the optical receiver to properly combine the signals received from various paths and the system performance that could be achieved by such processing. As a result of wideband optical technology it is reasonable to use multiple paths in fiber optic broadcast network to achieve simplicity and robustness. This leads to problems of how to reconcile signals received over multiple paths. The study examined the performance of fiber optics networks with incoherent sources and multiple paths. The networks were modeled as having either a fixed number of equal strength paths or an infinite number of paths with exponentially decreasing strengths.

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1 Introduction

Traditionally, communication networks have been designed so that only one path between a given transmitter and its intended receiver is used at a time. Multipath, or more than one simultaneous path between two communicating users, was considered bad because these paths would interfere with one another. Therefore, networks were structured so that only one path is used for a given communication.

With the new optical fiber technology and its very wide bandwidth, networks no longer have to be designed to conserve bandwidth. Instead, the excess bandwidth can be used to maintain several simultaneous, noninterfering paths between each pair of users in order to provide more robust communication. This is especially important in networks that are likely to have paths destroyed, military networks for example. The excess bandwidth allows the networks to be simpler and less expensive by having a transmitter broadcast its message over the entire network, eliminating the need for switching.

The idea of using broadcast networks with multiple paths leads to several issues that must be considered. One issue is how to best design topologies with multiple paths. Wasem [4] is studying this issue with respect to path strengths, and Ku [2] is studying this issue with respect to path lengths.

Apart from topological design, there is the issue of how to reconcile signals received over the multiple paths. A message sent by a source will propagate to its destination on many paths of different lengths. The destination will then receive many copies of this message delayed by different amounts of time. This is referred to as multipath interference. If more than one symbol is sent on this network then it is possible that data from different symbols will arrive at the destination at the same time, this is called intersymbol interference. The received signal containing multipath interference and intersymbol interference will then have to be processed to recover the original message.

This thesis looks at several types of receivers that can be used on such a network and evaluates their performance. It studies whether networks with multiple paths can give adequate performance and specifies any restrictions needed to obtain this performance.

Chapter 2 describes the network model that is used throughout this thesis. The source is modeled as an incoherent source emitting an infinite string of digital pulses. The network is modeled as a set of paths, each described by a strength and a delay. Networks are classified on the basis of path strength, and two networks, one with all paths having equal strengths and one with exponentially decreasing path strengths, are discussed.

The third chapter describes three receivers that can be used with the above networks; the zero-forcing equalizer, the minimum-mean-square-error equalizer, and the correlating receiver, and analyzes their performance with the above networks. The performance measure used is a signal-to-noise ratio where intersymbol interference is counted as noise. The zero-forcing equalizer is chosen because it is commonly used with networks containing intersymbol interference. The minimum-mean-square-error equalizer is chosen because it is the optimum receiver for the given performance measure. The correlating receiver is the receiver that would be optimum if there were no intersymbol interference and is anlayzed as a basis of comparison.

The results show that if all paths have equal strengths then only the minimum-mean-square-error equalizer gives adequate performance. However, if the path strengths are exponentially decreasing then the zero-forcing equalizer is adequate as well. As the differential delay between the paths becomes small with respect to the signaling interval, the amount of interference is reduced and all three receivers give adequate performance.

2 Network Model

The networks in consideration will have many users. It will be assumed that some type of multiplexing, such as frequency division, is used so that users do not interfere with each other. Therefore, only one communicating pair (source and destination) will be considered and the network will be modeled as a point-to-point channel between them.

2.1 Source Model

An interesting set of multipath broadcast networks are those with incoherent sources, specifically LED's. These are interesting because they are simple to build and avoid phase problems and synchronization problems that occur with coherent sources.

The LED has a very short coherence time. Since the autocorrelation function of its output is zero for time differences greater than the coherence time, it is effectively zero for all nonzero time differences. Therefore, it can be modeled as a source of power instead of as a source of EM fields. Its output can then be thought of as a stream of photons. It is a good assumption for an LED that the photons are emitted as a Poisson process.

The LED will be modulated by an infinite string of digital pulses. The photon emission rate will be proportional to:

$$\sum_{n=-\infty}^{\infty} s_{a_n}(t-nT)$$

where a_n is the value of bit n, either 0 or 1, $s_0(t)$ and $s_1(t)$ are the pulse shapes corresponding to 0 and 1, and T is the time between successive pulses.

2.2 Point-to-Point Channel Model

The general channel model has N paths between the source and the destination. The paths can include loops so there may be an infinite number of paths, in this case N would be infinity. Power is divided onto different paths by star couplers and the sum of the outputs of the different paths is incident upon the photodetector. The i^{th} path has a strength α_i associated with it. This path strength is equal to the fraction of source power that travels to the destination on that path or the fraction of signal photons that actually travel on that path. It is assumed that there are no optical amplifiers in the network. There may be excess loss due to the couplers. From conservation of power:

$$\sum_{i=1}^{N} \alpha_i \leq 1$$

Equality would imply no excess loss in the system. This will be referred to as the lossless case and will be used for the remainder of this thesis. This does not lose any generality because the analysis for the case with excess loss will be similar to the analysis presented here and will yield similar results. Networks of this type with excess coupler loss are discussed in [4].

The i^{th} path will have a time delay τ_i associated with it. For convenience the paths are numbered such that $\tau_{i+1} > \tau_i$. Multiple paths that have exactly the same length will be treated as one path with a strength equal to the sum of the strengths of the individual paths. It will be assumed that the differential delay between any two paths ($| \tau_i - \tau_j |$ for $i \neq j$) will be larger than the coherence time of the LED. Because of this, the outputs of the different paths will have zero covariance and the total average power incident on the detector will be the sum of the powers from the individual paths. Because of the high degree of randomness in the LED signal, the photon arrival rate will essentially equal that given by the average power. In certain situations that will be explained in chapter 3, it will be necessary to extend the constraint on the differential delay such that ($| \tau_i - \tau_j |$ for $i \neq j$) is larger than the reciprocal of the signal bandwidth. Topologies that obey these differential delay constraints are being studied in [2].

The channel can then be modeled as a linear system with impulse response:

$$h(t) = \sum_{i=1}^{N} \alpha_i \delta(t - \tau_i)$$

The noise is modeled as additive white Gaussian noise with spectral density $\frac{N_0}{2}$. The sources of this noise are white Gaussian thermal noise and Poisson shot noise. The shot noise comes from the signal light, the dark current, and background noise from users in other frequency bands. The shot noise is an inhomogeneous Poisson process with rate proportional to $P(t) + P_d + P_B$ where $P(t)$ is the signal power, P_d is the dark current equivalent power, and P_B is the power from the background noise.

The overall noise can be modeled as Gaussian in the case where the thermal noise is the dominant noise in the system. In the case where the shot noise is dominant, the noise at the output can be modeled as Gaussian if it has been passed through a sufficiently narrowband filter. This will occur if the bandwidth of the filter is much less than the rate of the shot noise.

The complete network model is shown in figure 1 where the received signal

$$r(t) = \sum_{n=-\infty}^{\infty} \sum_{i=1}^{N} \alpha_i s_{a_n}(t - \tau_i - nT) + n(t)$$

is the signal received at the destination before any processing is done on it.

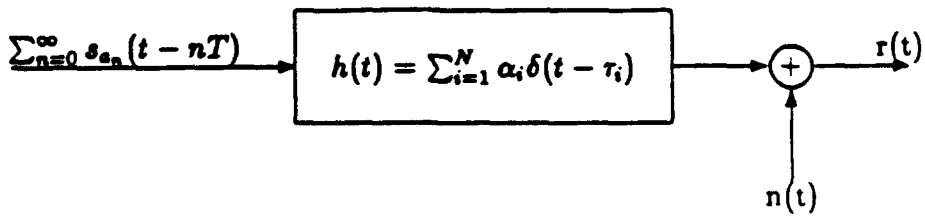

Figure 1: Network model

2.3 Networks of Interest

Networks will be considered base on classification of τ_i's and α_i's.

Initially, it will be assumed that the delays are all integer multiples of some delay τ. This is a reasonable assumption because in a real life situation a network could be assembled from fibers available in standard lengths. For this case:

$$\tau_i - \tau_j = (i-j)\tau \text{ for some } \tau > \text{ coherence time of LED}$$

This will be expressed as:

$$\tau_i = i\tau$$

For the path strengths α_i two different cases will be considered:

1. All paths have equal strength:

$$\alpha_i = \alpha \text{ for } i = 1 \text{ to } N$$

   where $\alpha = \frac{1}{N}$ in the lossless case.

2. Paths have exponentially decreasing strengths:

$$\alpha_i = \alpha^i \text{ for } i = 1 \text{ to } \infty$$

   where $\alpha = \frac{1}{2}$ in the lossless case.

According to [4], these two are reasonable and interesting assumptions.

3 Performance of Networks with Incoherent Sources

In this chapter, the performance of the networks described previously will be evaluated in combination with several different receivers. The first receiver to be considered is the zero-forcing equalizer which totally eliminates intersymbol interference (ISI) but may attenuate the signal and amplify the noise in the process. Next is the minimum-mean-square-error equalizer. This is the receiver that gives the best possible performance in terms of signal-to-noise ratio. Last, the correlator will be analyzed. The correlator is the receiver which would be optimum if only one bit were transmitted. It can be used as a basis for comparison.

The performance measure used to evaluate these receivers will be a signal-to-noise ratio. This is defined as:

$$SNR_0 = \frac{(E[\text{output of receiver due to signal}])^2}{var(\text{noise at output of receiver})} \tag{1}$$

The noise at the output of the receiver includes a component due to the additive noise in the system and possibly a component due to ISI. If the ISI is not completely eliminated by the receiver then the noise may not have a zero mean. This deterministic part of the noise can be subtracted from the output before any decision making is done. Therefore, the variance is the measure of interest.

Signaling with incoherent sources can use on-off-keying (OOK). In this chapter, the transmitted signal used will be

$$s_{a_n}(t) = \begin{cases} s(t) & \text{for } a_n = 1 \\ 0 & \text{for } a_n = 0 \end{cases}$$

where s(t) is the pulse shape and

$$Pr(a_n = 0) = Pr(a_n = 1) = \frac{1}{2}$$

3.1 The Zero Forcing Equalizer

The zero-forcing (ZF) equalizer is a filter that maximizes the signal-to-noise ratio subject to the constraint that intersymbol interference be eliminated at the digital sampling instants. Its response to a one bit signal will have zeros at all the sample times except the one corresponding to the signal bit. Therefore other bits in a series will not interfere at the sample time for a particular bit.

The signal-to-noise ratio for the zero-forcing equalizer which can be found in [1] is:

$$SNR_0 = \frac{2}{T^2 N_0} \frac{1}{\int_{\frac{-1}{2T}}^{\frac{1}{2T}} \frac{df}{\sum_{k=-\infty}^{\infty} \left|S(f-\frac{k}{T})\right|^2 \left|\sum_{i=1}^{N} a_i e^{-j2\pi\tau_i(f-\frac{k}{T})}\right|^2}} \tag{2}$$

This is obtained by using a filter with frequency response:

$$H_R(f) = \frac{\left[S(f)\sum_{i=1}^{N}\alpha_i e^{-j2\pi\tau_i}\right]^*}{\sum_{k=-\infty}^{\infty}\left|S(f-\frac{k}{T})\sum_{i=1}^{N}\alpha_i e^{-j2\pi\tau_i(f-\frac{k}{T})}\right|^2} \tag{3}$$

From equation 2 some general properties of these systems can be obtained. Suppose the differential delay between paths is very small, i.e. the limit as $\tau \to 0$, and assume that $T >> \tau$. Then the sum over the different paths becomes

$$lim_{\frac{T}{\tau}\to\infty}\left|\sum_{i=1}^{N}\alpha_i e^{-j2\pi\tau_i(f-\frac{k}{T})}\right|^2 = \left|\sum_{i=1}^{N}\alpha_i\right|^2$$

For the lossless case mentioned in chapter 2, this is equal to 1.

The signal to noise ratio expression is now

$$SNR_0 = \frac{2\left|\sum_{i=1}^{N}\alpha_i\right|^2}{T^2N_0}\frac{1}{\int_{\frac{-1}{2T}}^{\frac{1}{2T}}\frac{df}{\sum_{k=-\infty}^{\infty}\left|S(f-\frac{k}{T})\right|^2}} \tag{4}$$

Suppose the pulse shape $s(t)$ is chosen to have a flat spectrum over a bandwidth $\frac{m}{T}$ where m is a positive integer.

$$S(f) = \begin{cases} S_0 & |f| < \frac{m}{2T} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The pulse is chosen this way because it makes the shifted replicas in equation 2 simpler to analyze. This is not an actual pulse shape that could be used because this $S(f)$ gives a negative going $s(t)$ and the output of the LED must be a positive quantity. However, a small amount of power could be added to both $s_0(t)$ and $s_1(t)$ such that $s_1(t)$ is always positive. This deterministic amount of power could be subtracted from the output and the analysis would not be changed. Equation 4 now becomes

$$SNR_{MAX}^{m} = \frac{2mS_0^2\left|\sum_{i=1}^{N}\alpha_i\right|^2}{TN_0} \tag{6}$$

This is actually the maximum value of the signal-to-noise ratio. This is because in the limit $\frac{\tau}{T} \to 0$ all of the data from one bit will arrive at the destination before any of the data from the next bit. Therefore there will be no intersymbol interference. The maximum attainable signal-to- noise ratio is linearly proportional to $\frac{m}{T}$, the bandwidth of the transmitted pulse. Therefore, increasing the bandwidth will increase the best possible performance.

For the remainder of this section, the transmitted pulse will continue to have a flat spectrum over a finite bandwidth $\frac{m}{T}$ as in equation 5.

Equation 2 now simplifies to

$$SNR_0 = \frac{2S_0^2}{T^2 N_0} \frac{1}{\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{df}{\sum_{k=m\, terms} \left| \sum_{i=1}^{N} \alpha_i e^{-j2\pi\tau_i (f - \frac{k}{T})} \right|^2}} \tag{7}$$

For the case of an infinite number of exponential strength paths ($\alpha_i = \alpha^i$ for $i \geq 1$) with a bandwidth of $\frac{1}{T}(m = 1)$ this expression becomes

$$SNR_0 = \frac{2S_0^2}{TN_0} \frac{\alpha^2}{(1 + \alpha^2) - 2\alpha \frac{\sin\left(\frac{\pi\tau}{T}\right)}{\frac{\pi\tau}{T}}} \tag{8}$$

This attains its maximum when there is no intersymbol interference which is when $\tau \rightarrow 0$. This is equivalent to having only one path. In this limit equation 8 becomes:

$$SNR_{MAX}^1 = \frac{2S_0^2}{TN_0} \frac{\alpha^2}{(1 - \alpha)^2} \tag{9}$$

where the subscript 1 stands for $m = 1$. Figure 2 shows a plot of the normalized signal-to-noise ratio, $\frac{SNR_0}{SNR_{MAX}^1}$, for the lossless case of $\alpha = \frac{1}{2}$. Since the bandwidth is a function of the data rate this curve can be viewed as a function of the differential delay of a system with constant bandwidth and data rate.

As the differential delay gets small with respect to the signaling interval $\left(\frac{\tau}{T} \rightarrow 0\right)$ the performance approaches the maximum value. This is because the data from a particular bit will arrive on many paths long before the data from the next bit will start arriving. Therefore, there is less intersymbol interference and the performance is better.

In the region where the differential delay is on the order of the signaling interval or smaller, there are oscillations in the curve. This is because in this region the amount of intersymbol interference is a function of how much bits from different paths overlap each other. In the case where τ is an integer multiple of T, they completely overlap. This is not the worst case because the paths have exponentially decreasing strengths. The worst case will be shown subsequently. At these points the expression becomes:

$$SNR_0 = \frac{2S_0^2}{TN_0} \frac{\alpha^2}{1 + \alpha^2} \quad \text{for } \tau = lT \quad l = 1, 2, \ldots \tag{10}$$

This is also the value in the limit $\frac{\tau}{T} \rightarrow \infty$ which is the case of an infinite differential delay which is similar to only one path. This is different from the signal-to-noise ratio expected from a one path system because it is still a multipath system and there will still be intersymbol interference.

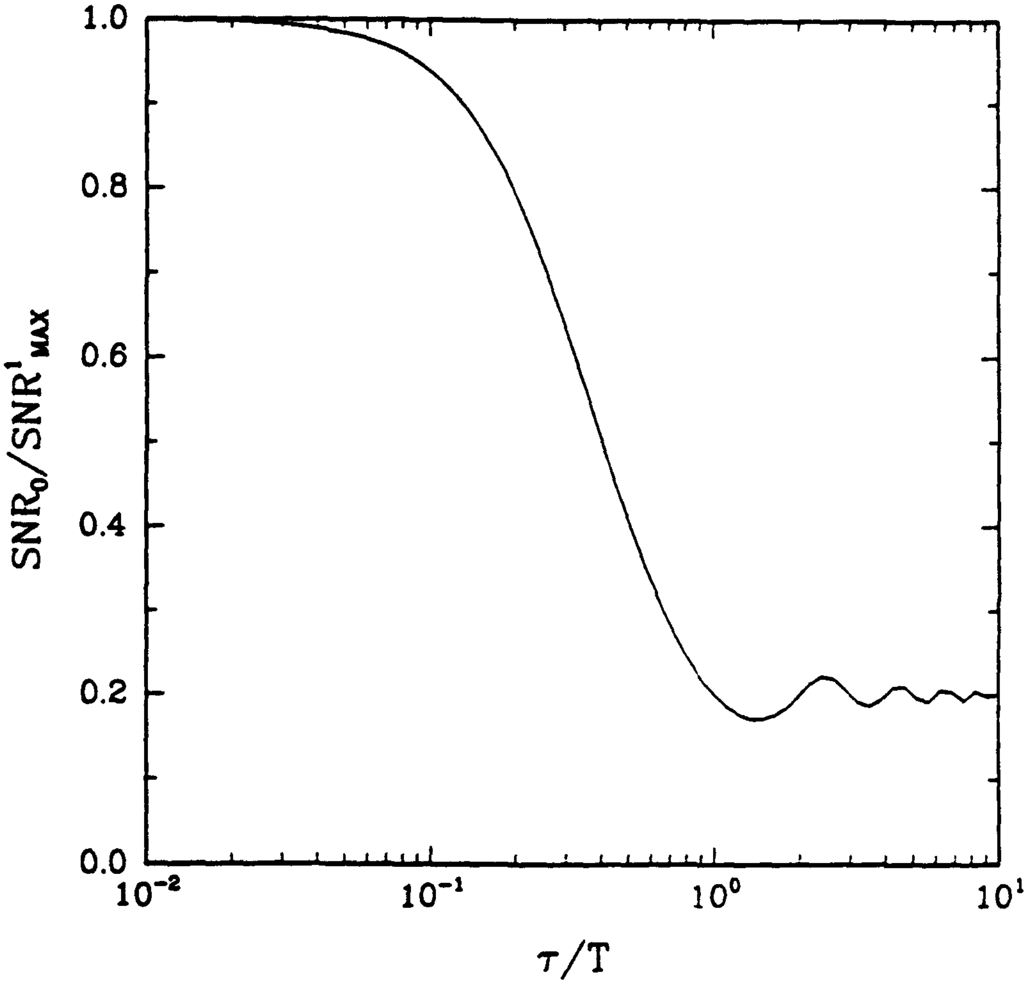

Figure 2: Performance of the zero-forcing equalizer in a system with exponentially decreasing path strengths and a signal bandwidth of $\frac{1}{T}$

An interesting fact about this curve is that it has a nonzero minimum performance for all values of $\frac{\tau}{T}$. This means that a user can conceivably build a network with any differential delay (τ) and operate it at any data rate (T). By differentiating equation 8 and setting it equal to zero, it can be found that all local minima and maxima occur at points where:

$$tan\left(\frac{\pi\tau}{T}\right) = \frac{\pi\tau}{T}$$

The absolute minimum occurs when $-2\alpha\frac{\sin\left(\frac{\pi\tau}{T}\right)}{\frac{\pi\tau}{T}}$ is its largest. This occurs when $\frac{\tau}{T} = 1.43$ and this minimum value is

$$SNR_{MIN} = \frac{2S_0^2}{TN_0}\frac{\alpha^2}{1+\alpha^2+0.434\alpha} \tag{11}$$

If the bandwidth is now increased to $\frac{m}{T}$ where m is an integer then equation 7 becomes:

$$SNR_0 = \frac{2S_0^2}{TN_0}\frac{\alpha^2}{T\int_{\frac{-1}{2T}}^{\frac{1}{2T}}\frac{df}{\sum_{k=m\,terms}\left[1+\alpha^2-2\alpha\cos\left(2\pi\tau\left(f-\frac{k}{T}\right)\right)\right]^{-1}}} \tag{12}$$

This attains the same maximum value at $\frac{\tau}{T}\rightarrow 0$ for any value of m. At this limit:

$$\lim_{\frac{T}{\tau}\rightarrow\infty}\cos\left(2\pi\tau\left(f-\frac{k}{T}\right)\right)\rightarrow\cos(2\pi\tau f)$$

and each of the m terms in the sum are the same. The expression then becomes:

$$\begin{aligned} SNR_{MAX}^{m} &= \frac{2S_0^2 m}{TN_0}lim_{\frac{\tau}{T}\rightarrow 0}\frac{\alpha^2}{T\int_{\frac{-1}{2T}}^{\frac{1}{2T}}(1+\alpha^2-2\alpha\cos 2\pi\tau f)df} \\ &= \frac{2S_0^2 m}{TN_0}\frac{\alpha^2}{(1-\alpha)^2} \end{aligned} \tag{13}$$

When m=1 this reduces to equation 9, the maximum for a bandwidth of $\frac{1}{T}$.

An interesting characteristic of equation 12 is that at the points where τ is an integer multiple of T the value of the signal-to-noise ratio is independent of τ. At these points

$$\cos\left(2\pi\tau f - k\frac{2\pi\tau}{T}\right) = \cos(2\pi\tau f)$$

All the terms in the sum over k are now the same and the expression becomes:

$$SNR_0 = \frac{2S_0^2 m}{TN_0}\frac{\alpha^2}{1+\alpha^2} \quad \text{for } \tau = lT \quad l = 1,2,\ldots \tag{14}$$

This is the same as equation 10 which is just the $m = 1$ case. Furthermore, as m gets large, the minimum value of the curve occurs arbitrarily close to these points. This can be observed from

plots of these curves. By taking the derivative and setting it equal to zero it can be shown that these are the actual minima in the limit $m \rightarrow \infty$.

The integral in equation 12 can be evaluated numerically. Figure 3 shows a plot of this expression for $m = 10$ normalized to the maximum for $m = 1$. This can be compared to figure 2, which has a smaller bandwidth. At small values of $\frac{\tau}{T}$ (small differential delay) the larger bandwidth gives better performance by a factor of m. At large values of $\frac{\tau}{T}$ both have nonzero performance. The curve with the smaller bandwidth retains its maximum value until a larger value of $\frac{\tau}{T}$.

Figure 4 shows a plot of the performance vs. m (bandwidth) of the same system for several values of $\frac{\tau}{T}$. It can be seen when τ is an integer multiple of T the curve is a straight line depending only on m. This has been shown previously in equation 14. It can also be observed that when τ is very small ($\tau = 10^{-4}T$) the performance is essentially its maximum value for all values of m shown.

For the case of N equal strength paths, equation 7 becomes:

$$SNR_0 = \frac{2S_0^2}{TN_0} \frac{\alpha^2}{T \int_{\frac{-1}{2T}}^{\frac{1}{2T}} \frac{df}{\sum_{k=m\,terms} \frac{1-\cos\left(2\pi\tau N\left(f-\frac{k}{T}\right)\right)}{1-\cos\left(2\pi\tau\left(f-\frac{k}{T}\right)\right)}}} \tag{15}$$

This will have zeros at the points where $\frac{\tau}{T}$ is an integer greater than 2. This is because at these points the data bits from different paths will exactly overlap each other. Since the paths all have the same strength there is no way to determine which data bit came from which path. Therefore, while forcing the ISI to zero, it will force the signal to zero as well.

Equation 15 will attain its maximum in the limit $\frac{\tau}{T} \rightarrow 0$.

$$SNR_{MAX}^{m} = \frac{2S_0^2 m}{TN_0} N^2 \alpha^2 \tag{16}$$

Figure 5 shows the normalized performance for $N = 10$ equal strength paths and a bandwidth of $\frac{10}{T}$. It is normalized to the $m = 1$ maximum for this system. In the region of small $\frac{\tau}{T}$ there is little or no ISI and the performance is close to its maximum. In the region of higher ISI (larger differential delay) there are many zeros. If $\frac{\tau}{T}$ cannot be adjusted very precisely or if the system will be used with different data rates then this performance will not be acceptable and a different type of receiver will be needed.

If the signal bandwidth is $\frac{1}{T}(m = 1)$, then equation 15 simplifies to

$$SNR_0 = \frac{2S_0^2\alpha^2}{T^2N_0} \frac{1}{\int_{\frac{-1}{2T}}^{\frac{1}{2T}} \frac{1-\cos(2\pi\tau f)}{1-\cos(2\pi\tau N f)} df} \tag{17}$$

The integral in the denominator of this equation will blow up for $\frac{\tau}{T} > \frac{2}{N}$ and the performance will be zero in this region. Therefore a zero-forcing equalizer cannot be used for a small bandwidth

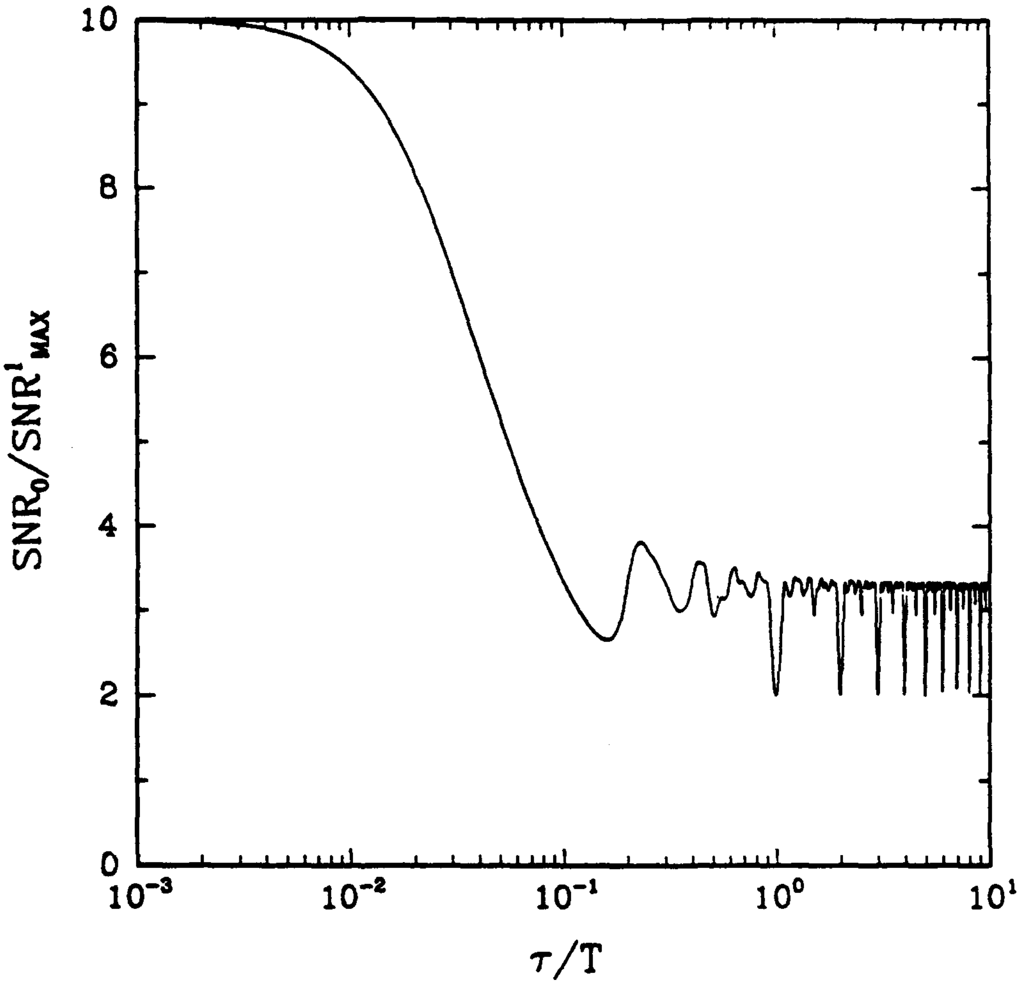

Figure 3: Performance of the zero-forcing equalizer in a system with exponentially decreasing path strengths and a signal bandwidth of $\frac{10}{T}$.

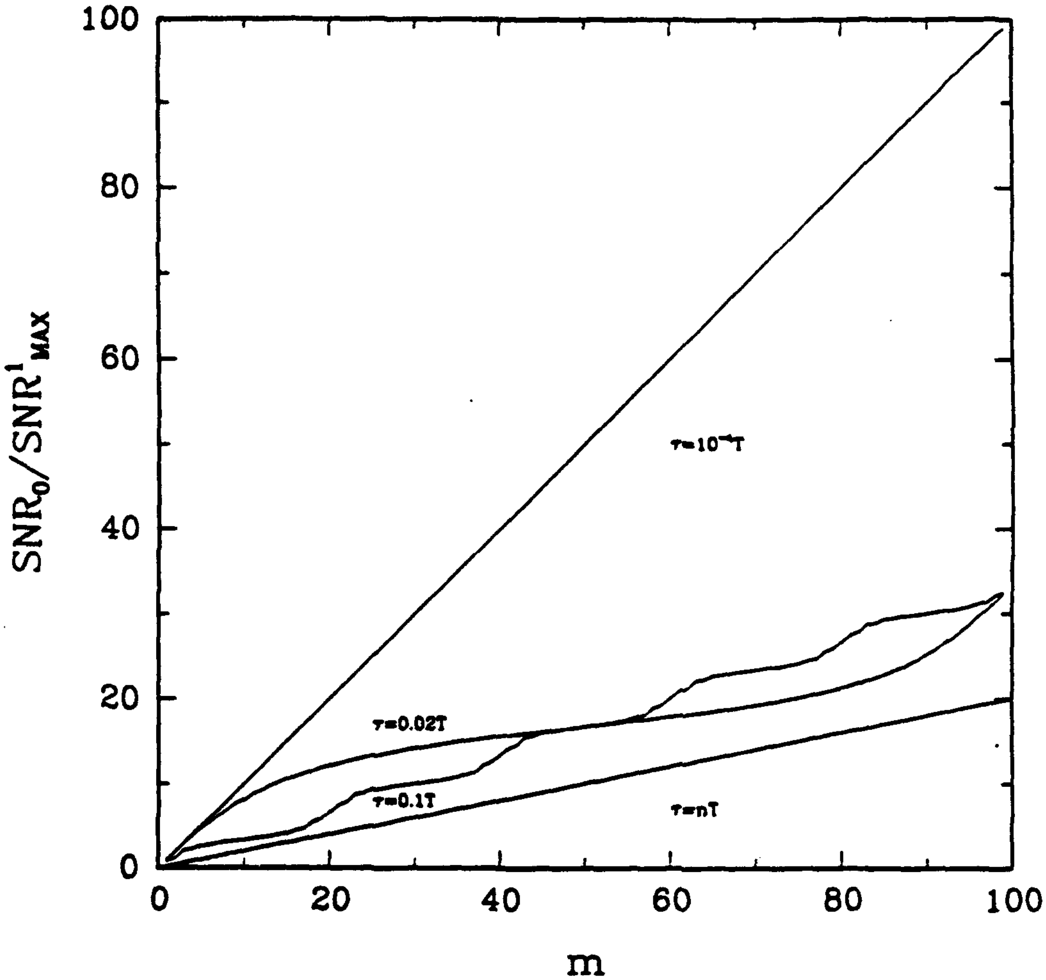

Figure 4: Performance vs. bandwidth of the zero-forcing equalizer in a system with exponentially decreasing path strengths for several values of τ.

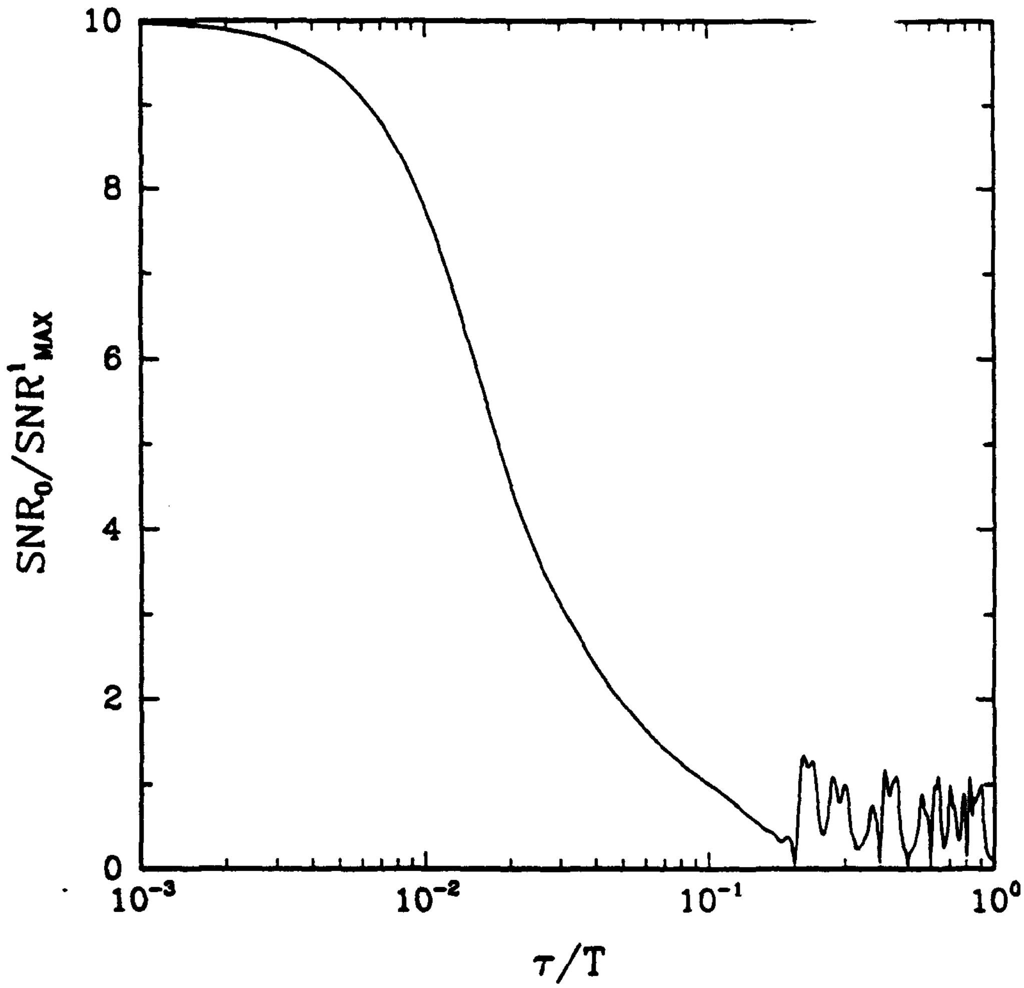

Figure 5: Performance of the zero-forcing equalizer in a system with 10 equal strength paths and a signal bandwidth of $\frac{10}{T}$

signal ($m = 1$) on a network with equal strength paths and large differential delay (greater than $\frac{2T}{N}$).

3.2 Minimum Mean Square Error Equalizer

One disadvantage of the ZF equalizer is it tries to absolutely force zeros rather than just get good performance and may amplify the noise in the process. In contrast with this, the minimum-mean-square-error (MMSE) equalizer balances the effect of intersymbol interference with the amplification of noise by minimizing the mean-square-error. By definition, the MMSE equalizer is the optimum receiver when signal-to-noise ratio is the performance measure.

The performance for the MMSE equalizer is described by Proakis in [3]. There he defines a performance index J which is the expected value of the squared error. He then minimizes it to find J_{MIN} which for the system here is:

$$J_{MIN} = T\frac{N_0}{2}\int_{\frac{-1}{2T}}^{\frac{1}{2T}} \frac{df}{\frac{1}{T}\sum_{k=-\infty}^{\infty}\left|S(f-\frac{k}{T})\right|^2\left|\sum_{i=1}^{N}\alpha_i e^{-j2\pi\tau_i(f-\frac{k}{T})}\right|^2+\frac{N_0}{2}} \tag{18}$$

The signal-to-noise ratio is then:

$$SNR_0 = \frac{1-J_{MIN}}{J_{MIN}} \tag{19}$$

Again assume the transmitted pulse has a flat spectrum over a finite bandwidth as in equation 5. Now the equation for J_{MIN} becomes

$$J_{MIN} = \frac{TN_0}{2}\int \frac{df}{\frac{S_0^2}{T}\sum_{|k|<\frac{m}{2}}\left|\sum_{i=1} N\alpha_i e^{-j2\pi\tau_i(f-\frac{k}{T})}\right|^2+\frac{N_0}{2}} \tag{20}$$

As with the ZF equalizer, this has its maximum in the limit $\frac{\tau}{T} \rightarrow 0$, where there is no intersymbol interference. In this limit:

$$\lim_{\frac{\tau}{T}\rightarrow 0} J_{MIN} = \frac{1}{\frac{2S_0^2 m}{TN_0}\left|\sum_{i=1}^{N}\alpha_i\right|^2+1}$$

and the maximum performance becomes:

$$SNR_{MAX}^{m} = \frac{2S_0^2 m}{TN_0}\left|\sum_{i=1}^{N}\alpha_i\right|^2$$

This is the same as the maximum for the zero-forcing equalizer. This is true because when there is no ISI these two receivers are essentially the same. Figure 6 shows the performance of the MMSE equalizer for ten equal strength paths and a bandwidth of $\frac{10}{T}$. This can be compared to figure 5 which is the same system with a ZF equalizer. Over the region where $\frac{\tau}{T}$ is large the MMSE has

more constant performance and is less sensitive to small changes in $\frac{\tau}{T}$ than the ZF equalizer. In the region where $\frac{\tau}{T}$ is small the two have pretty much equivalent performance.

Figure 7 shows the performance of an MMSE equalizer for exponential strength paths and a bandwidth of $\frac{10}{T}$. It can be compared to figure 3 which is the performance of the same system with a ZF equalizer. The normalized performances for these two cases are very similar and can be interpreted the same way.

3.3 The Correlating Receiver

The correlator is the receiver that would be optimum if only the $n = 0$ bit were transmitted. It eliminates the multipath interference of a single bit on itself but does not eliminate ISI at all. It is included here as a basis of comparison and to illustrate why the equalizers are necessary to eliminate ISI. The following is a performance analysis of this receiver when it receives an infinite string of bits.

The correlator can be implemented by correlating the received signal $r(t)$ with:

$$g(t) = \sum_{i=1}^{N} \alpha_i s(t - \tau_i)$$

where the α_i's and τ_i's are the same as in $h(t)$. The received signal $r(t)$ is passed through the correlator and the output which will be called X is

$$X = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} \alpha_i\alpha_j \int s_{a_n}(t - \tau_i - nT)s(t - \tau_j)dt + W \tag{21}$$

where W is the term resulting from the noise:

$$W = \int n(t) \sum_{i=1}^{N} \alpha_i s(t - \tau_i)dt$$

Assuming OOK is used, this signal can be expressed as

$$X = \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i\alpha_j \int a_0 s(t - \tau_i)s(t - \tau_j)dt$$
$$+ \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i\alpha_j \int a_n s(t - \tau_i - nT)s(t - \tau_j)dt + W$$

where a_0 is the desired information symbol. If the differential delay condition stated in Chapter 2 is extended such that the differential delay is larger than the reciprocal of the signal bandwidth, then it is possible to find a pulse shape $s(t)$ such that

$$\int s(t - \tau_i)s(t - \tau_j)dt = 0 \qquad \text{for } i \neq j$$

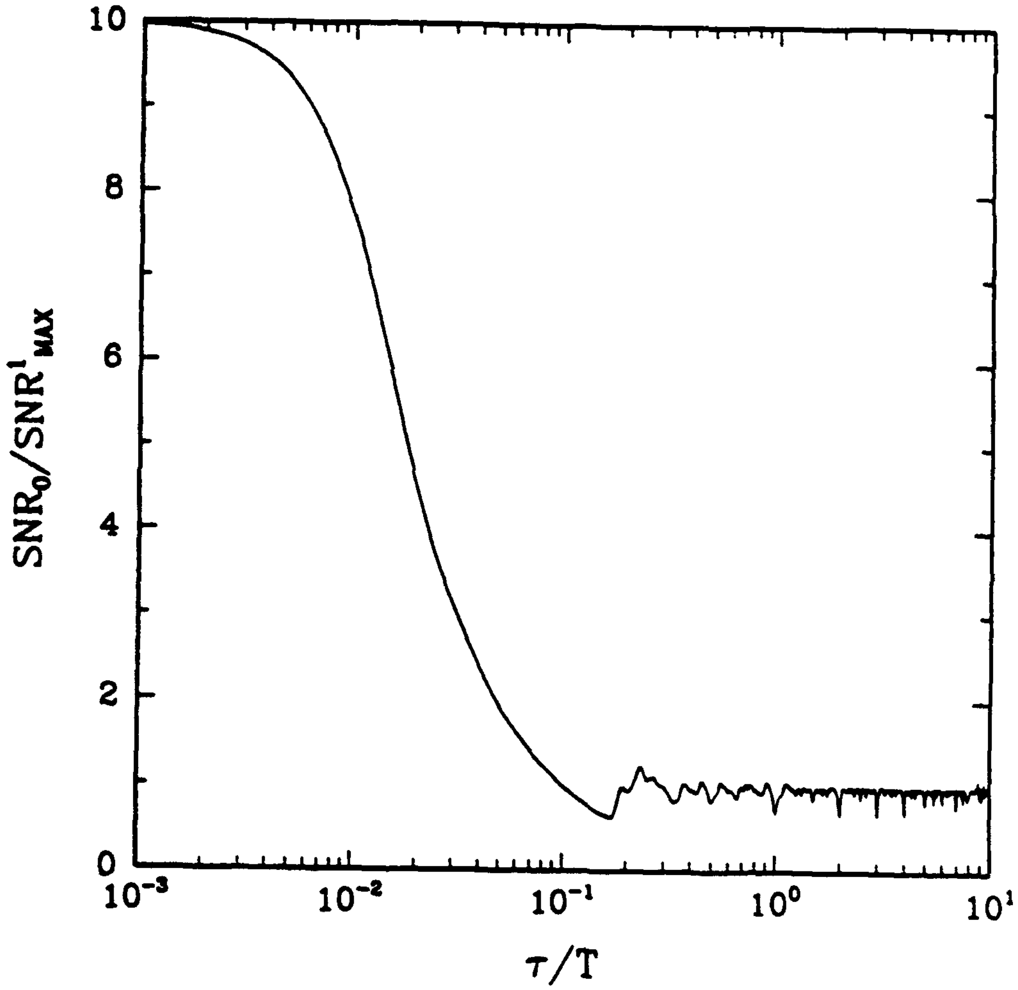

Figure 6: Performance of the MMSE equalizer in a system with 10 equal strength paths and a signal bandwidth of $\frac{10}{T}$.

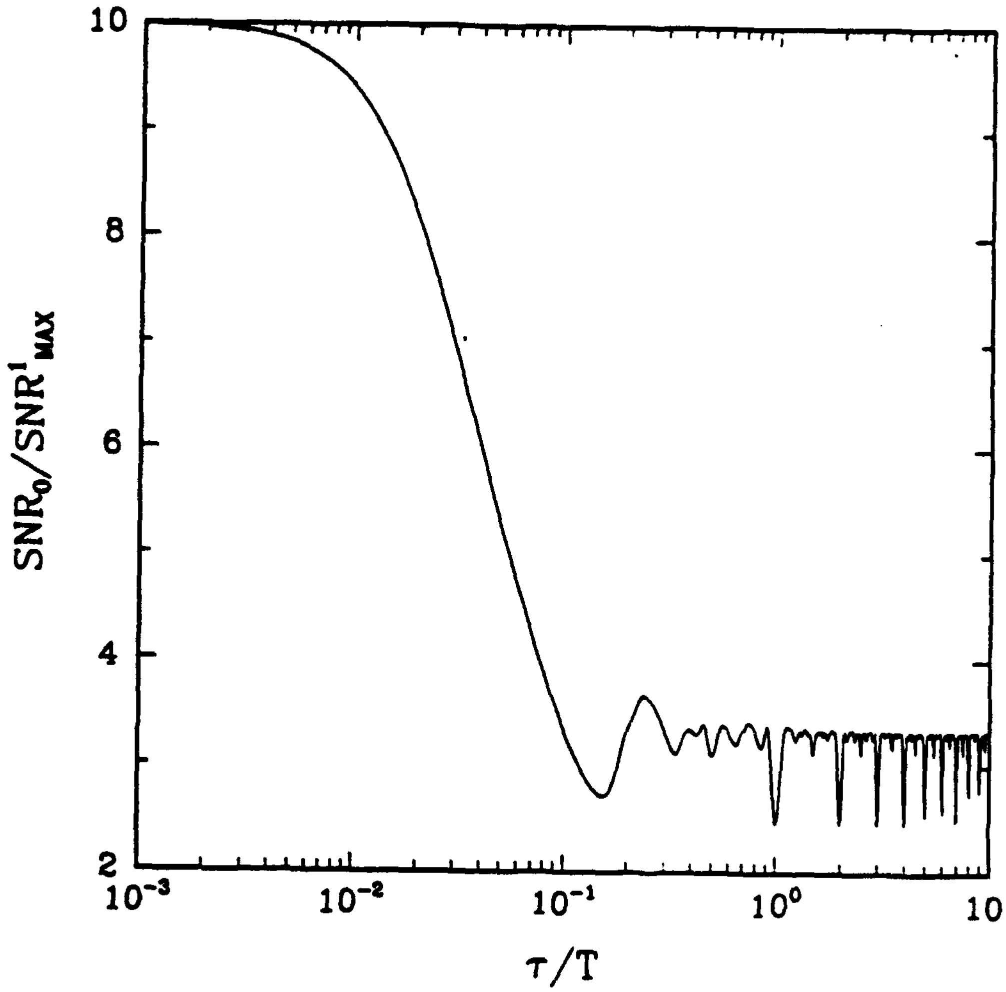

Figure 7: Performance of the MMSE equalizer in a system with exponentially decreasing path strengths and a signal bandwidth of $\frac{10}{T}$

The choice of $s(t)$ that will be used in this section will have a bandwidth equal to $\frac{m}{\tau}$ for $m = 1, 2, 3 \ldots$ and have a flat spectrum in that region.

$$S(f) = \begin{cases} S_0 & \frac{-m}{2\tau} < f < \frac{m}{2\tau} \\ 0 & \text{otherwise} \end{cases}$$

This is chosen for simplicity and has the property:

$$\int s(t - \tau_i) s(t - \tau_j) dt = 0 \qquad \text{for } i \neq j$$

This choice of $s(t)$ eliminates the multipath interference between shifted versions of bit $n = 0$ and leaves just the intersymbol interference (from other bits) and the noise to contend with. This simplifies the expression for X to:

$$X = \sum_{i=1}^{n} \alpha_i^2 \int a_0 s^2(t) dt + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j \int a_n s(t - \tau_i - nT) s(t - \tau_j) dt + W$$

This can be separated into

$$X = X_S + X_{ISI} + W \tag{22}$$

where X_S is the term resulting from the signal:

$$X_S = \sum_{i=1}^{N} \alpha_i^2 a_0 \int s^2(t) dt \tag{23}$$

and X_{ISI} is the term resulting from the intersymbol interference:

$$X_{ISI} = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j a_n \int s(t - \tau_i - nT) s(t - \tau_j) dt \tag{24}$$

This can be written as:

$$X_{ISI} = \frac{S_0^2 m}{\tau} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} a_n C_n \tag{25}$$

where

$$\begin{aligned} C_n &= \frac{\tau}{S_0^2 m} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j \int s(t - \tau_i - nT) s(t - \tau_j) dt \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j \frac{\sin\left[2\pi m\left(i - j + \frac{nT}{\tau}\right)\right]}{2\pi m\left(i - j + \frac{nT}{\tau}\right)} \end{aligned} \tag{26}$$

is proportional to the ISI from the n^{th} bit.

The noise term W is Gaussian noise because it consists of white Gaussian noise $n(t)$ passed through a linear system. W has mean 0 and variance:

$$
\begin{aligned}
\sigma_W^2 &= E(W^2) \\
&= E[n^2(t)] \left\{ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \alpha_i \alpha_j \int s(t-\tau_i) s(t-\tau_j) dt + \sum_{i=1}^{N} \alpha_i^2 \int s^2(t-\tau_i) dt \right\}
\end{aligned}
$$

The first term is zero by choice of $s(t)$ and:

$$\int s^2(t-\tau_i) dt = \int s^2(t) dt = \frac{S_0^2 m}{\tau}$$

Therefore

$$\sigma_W^2 = \frac{N_0 S_0^2 m}{2\tau} \sum_{i=1}^{N} \alpha_i^2$$

The signal-to-noise ratio for the correlator is defined as

$$(SNR)_0 = \frac{(E[X_S])^2}{var(X_{ISI}) + \sigma_W^2}$$

From equations 23 and 25:

$$E[X_S] = \frac{S_0^2 a_0 m}{2\tau} \sum_{i=1}^{N} \alpha_i^2$$

$$var(X_{ISI}) = \left(\frac{S_0^2 a_0 m}{2\tau} \right)^2 \sum_{\substack{n=-\infty \\ n \neq 0}}^{n=\infty} C_n^2$$

This gives the performance expression:

$$SNR_0 = \frac{\frac{S_0^2 a_0^2 m}{2N_0 \tau} \left(\sum_{i=1}^{N} \alpha_i^2 \right)^2}{\frac{S_0^2 a_0^2 m}{2N_0 \tau} \sum_{\substack{n=-\infty \\ n \neq 0}}^{n=\infty} C_n^2 + \sum_{i=1}^{N} \alpha_i^2} \tag{27}$$

where the C_n are given by equation 26. This is a function of $\frac{T}{\tau}$ since the bandwidth of the pulse is a function of τ rather than T. It has its maximum at the point when the intersymbol interference term is zero. This occurs at values of $\frac{T}{\tau}$ for which all $C_n = 0$. From equation 26 it is obvious that these occur at

$$\frac{T}{\tau} = l + \frac{1}{2} \quad \text{where } l = 0, 1, 2, \ldots \quad \text{except } j - i = n(l + \frac{1}{2})$$

The maximum signal-to-noise ratio is then:

$$SNR_{MAX}^{m} = \frac{S_0^2 a_0^2 m}{2N_0 \tau} \sum_{i=1}^{N} \alpha_i^2$$

This is also the signal-to-noise ratio of only the $n = 0$ bit is transmitted.

This leads to the normalized performance expression:

$$\frac{SNR_0}{SNR^1_{MAX}} = \frac{m\sum_{i=1}^{N}\alpha_i^2}{\frac{S_0^2a_0^2m}{2N_0\tau}\sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty} C_n^2 + \sum_{i=1}^{N}\alpha_i^2} \tag{28}$$

From this expression one can analyze the correlator's performance. In the limit $\frac{T}{\tau} \to \infty$, which corresponds to only one bit transmitted over all time, the $C_n \to 0$ and the expression will improve. This makes sense because if there is only one bit then there will be no intersymbol interference. In the limit $\frac{T}{\tau} \to 0$, which corresponds to an infinite data rate, the performance goes to zero.

Figures 8 through 14 show the normalized performance expression (equation 28 plotted versus $\frac{T}{\tau}$). Assuming τ, the differential delay, is a fixed parameter of the network, this can be viewed as a plot of performance versus data rate where small T corresponds to a high data rate and $T \to \infty$ corresponds to only one bit being sent for all time.

Figures 8 through 12 show the correlator performance for ten equal strength paths. For simplicity the network is assumed to be lossless so

$$\alpha_i = \frac{1}{10} \quad \text{for } i = 1 \text{ to } 10$$

Figure 8 has the constant $\frac{S_0^2a_0^2}{2N_0\tau} = 1$ and a bandwidth of $\frac{10}{\tau}$. If the user is willing to operate at a data rate slow enough that $\frac{T}{\tau} > 10$ then the performance will be close to the maximum available from the network and the correlator is an acceptable receiver. If the user wants to operate at a high rate such that $\frac{T}{\tau} < 10^{-2}$ then the performance becomes unacceptable and a different type of receiver must be used. In the region $10^{-1} < \frac{T}{\tau} < 10$ the performance curve has wild oscillations. A user can operate in this region with good performance only if $\frac{T}{\tau}$ can be set very precisely. The points where $\frac{T}{\tau} = l + \frac{1}{2}$ for some integer l are the optimum points, but from observation of the graph it is clear that other "good" points exist depending on the user's definition of good. It is highly unlikely that $\frac{T}{\tau}$ can be adjusted precisely enough to guarantee good performance in this region.

If the use has a required data rate it may be possible to design the network with τ small enough that the required data rate is in the acceptable operating region.

Figures 9 and 10 illustrate the performance of this same system with different values of $\frac{S_0^2a_0^2}{2N_0\tau}$. Figure 9 is somewhat deceptive. In this figure, the constant $\frac{S_0^2a_0^2}{2N_0\tau} = 0.01$. This corresponds to a system where the system noise, N_0, is dominant over the ISI. For this reason the oscillations in the curve are quite small. However, since the maximum value is proportional to this constant, the maximum value here is much smaller than in figure 8. Figure 10 corresponds to a system with the constant $\frac{S_0^2a_0^2}{2N_0\tau} = 100$, in this case the ISI dominates over the system noise and the regions of good and bad performance are quite distinct.

Figure 11 shows the same system with a much larger bandwidth of $\frac{100}{\tau}$. This allows the user to operate at much higher data rates (smaller T) if $\frac{T}{\tau}$ can be specified precisely. The region where

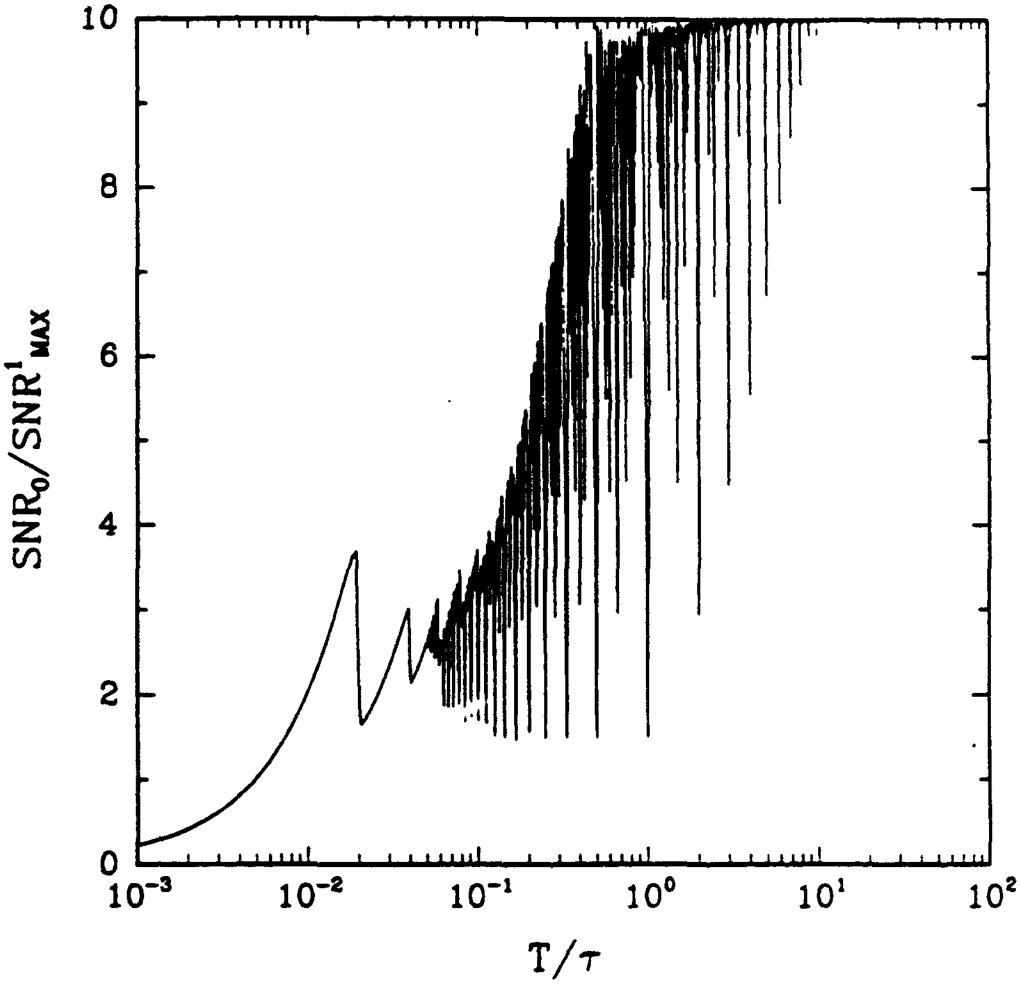

Figure 8: Performance of the correlating receiver in a system with 10 equal strength paths, a signal bandwidth of $\frac{10}{T}$, and the constant $\frac{S_0^2 a_0^2}{2N_0 \tau} = 1$

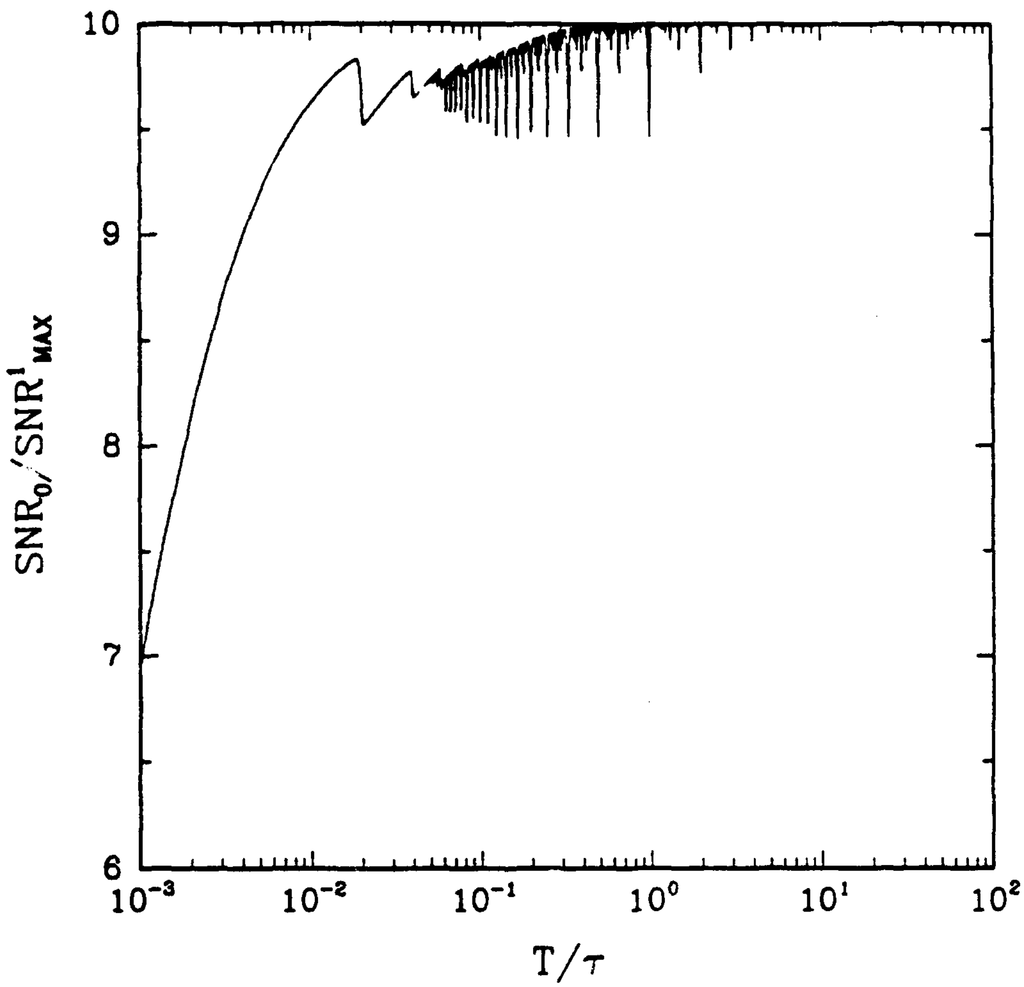

Figure 9: Performance of the correlating receiver in a system with 10 equal strength paths, a signal bandwidth of $\frac{10}{T}$, and the constant $\frac{S_0^2 a_0^2}{2N_0\tau} = 0.01$

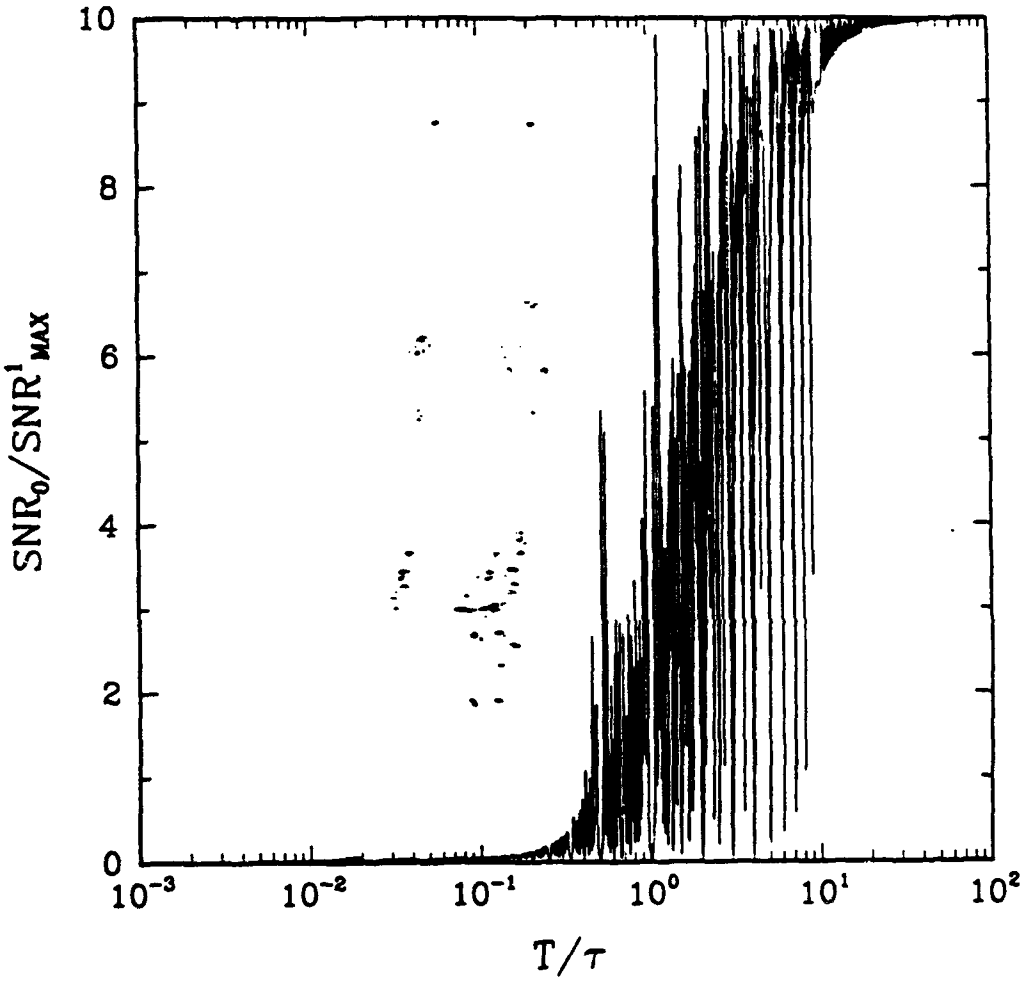

Figure 10: Performance of the correlating receiver in a system with 10 equal strength paths, a signal bandwidth of $\frac{10}{T}$, and the constant $\frac{S_0^2 a_0^2}{2N_0\tau} = 100$

performance approaches its maximum is only slightly improved over the bandwidth equal to $\frac{10}{\tau}$ case.

Figure 12 shows the performance for a smaller bandwidth of $\frac{2}{\tau}$. Here the oscillations have decreased but so has the maximum performance.

If the number of paths is decreased as in figure 13 (where $N = 2$ paths) the normalized signal to noise ratio approaches unity at a lower value of $\frac{T}{\tau}$. This is because with less paths there is less intersymbol interference. For example, if there are only 2 paths then at a given sample time, there can only be interfering data from one other bit. C_n, which is the intersymbol interference from the n^{th} bit, is proportional to N^2 while the other terms in equation 27 are proportional to N. Although decreasing the number of paths improves performance it decreases the robustness of the system, which is the reason for having multiple paths.

Figure 14 shows the performance of the correlator for exponential path strengths, $\alpha_i = \alpha^i$. Here the lossless case of $\alpha = \frac{1}{2}$ is used. In this plot $\frac{S_0^2 a_0^2}{2N_0} = 1$ and the bandwidth equals $\frac{10}{\tau}$. This has less intersymbol interference than equal strength paths which is shown by smaller oscillations in the curve. This is because one path is much stronger than the others. A data rate corresponding to $\frac{T}{\tau} > 1$ will give good performance on this system as will higher data rates if $\frac{T}{\tau}$ can be adjusted precisely.

According to [], real networks are a *combination of equal strength* and exponentially decreasing strength paths. Therefore, if a correlating receiver is used, the performance would more closely reflect the equal strength path case which has the worse performance of the two.

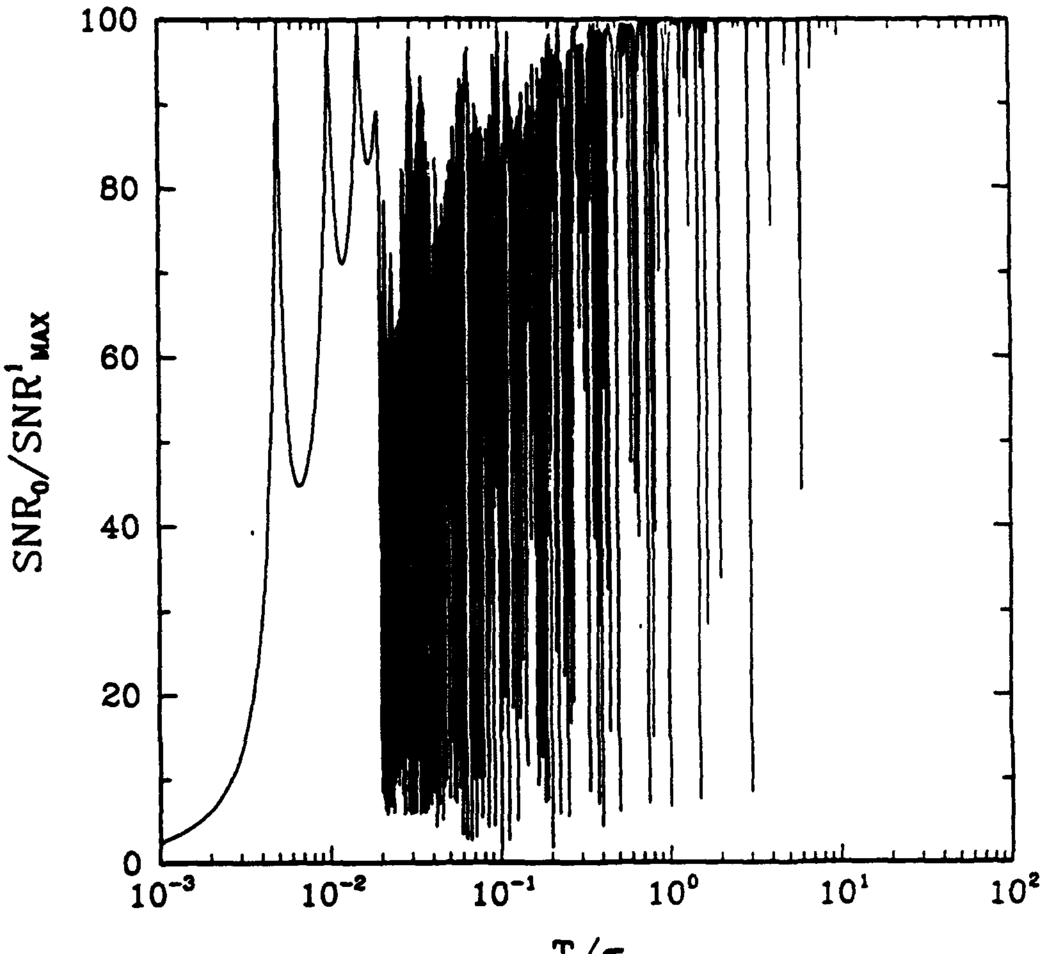

Figure 11: Performance of the correlating receiver in a system with 10 equal strength paths, a signal bandwidth of $\frac{100}{T}$, and the constant $\frac{S_0^2 a_0^2}{2N_0\tau} = 1$

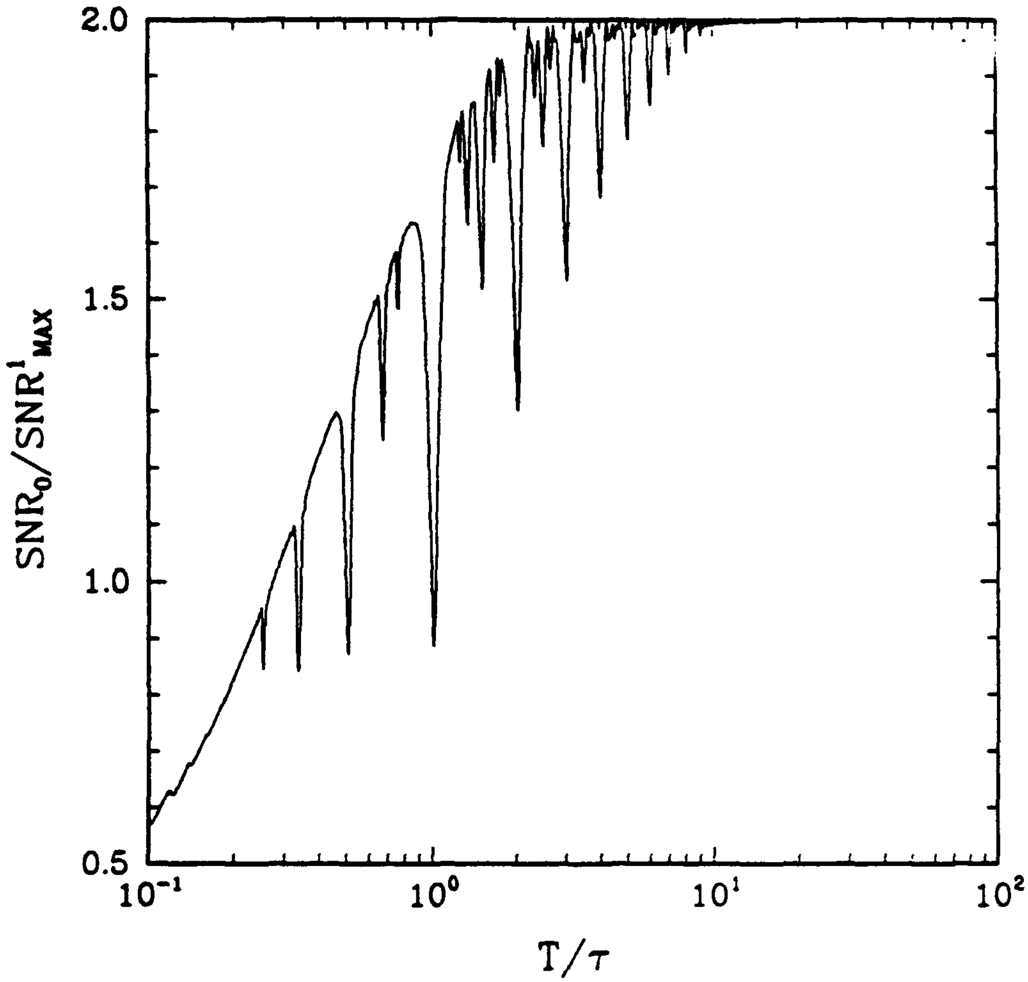

Figure 12: Performance of the correlating receiver in a system with 10 equal strength paths, a signal bandwidth of $\frac{2}{T}$, and the constant $\frac{S_0^2 a_0^2}{2N_0\tau} = 1$

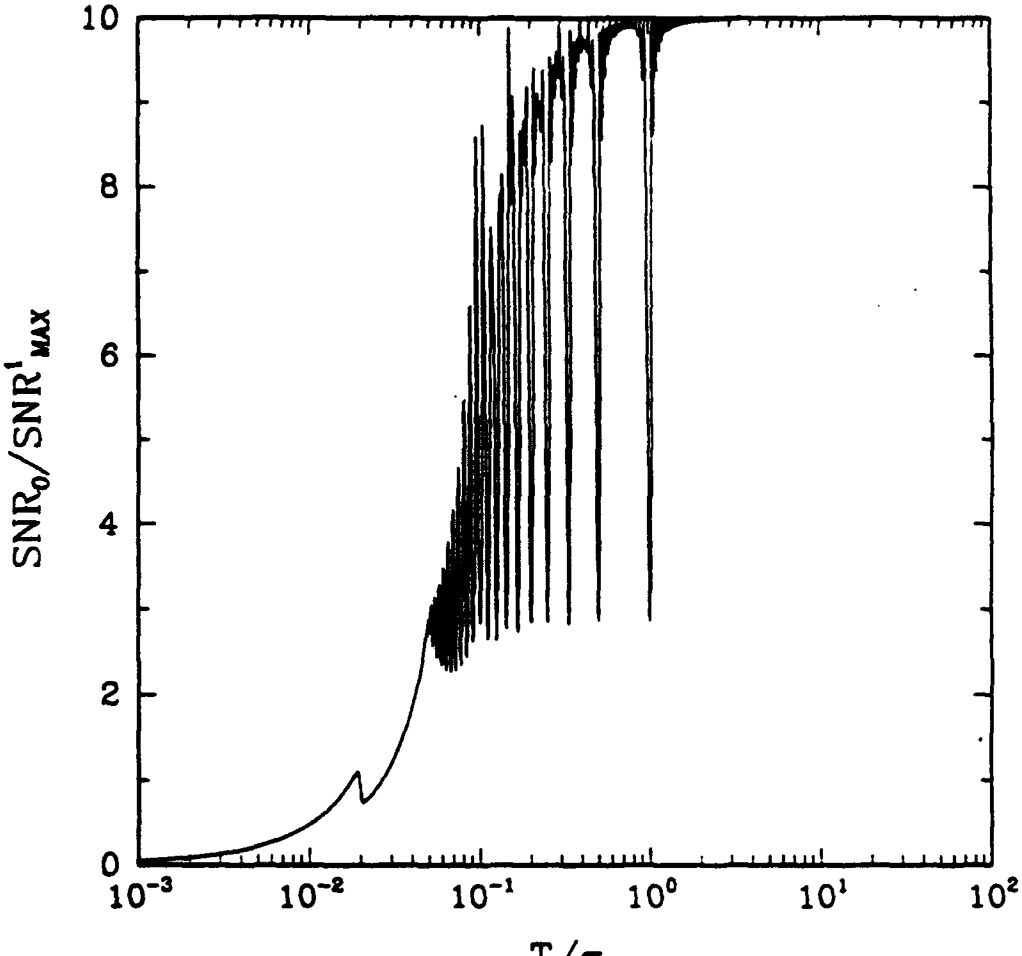

Figure 13: Performance of the correlating receiver in a system with 2 equal strength paths, a signal bandwidth of $\frac{10}{T}$, and the constant $\frac{S_0^2 a_0^2}{2N_0\tau} = 1$

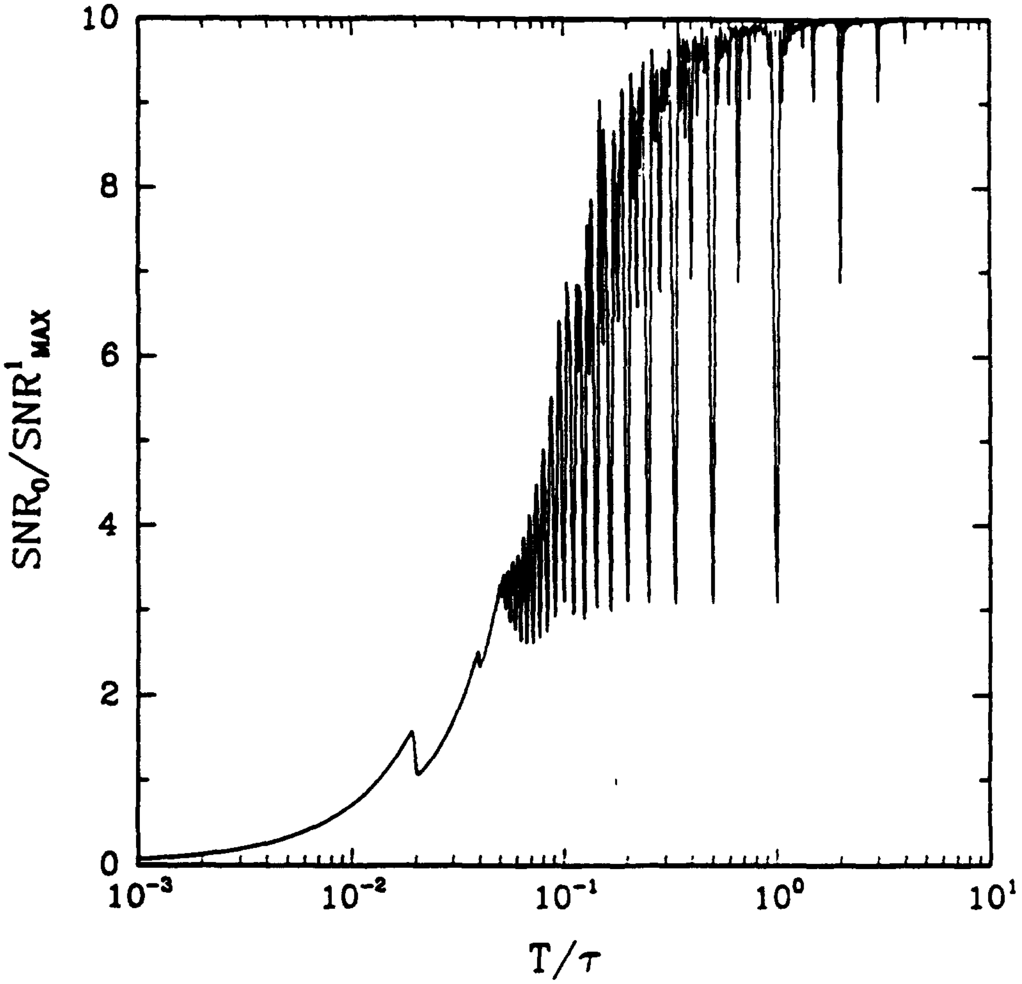

Figure 14: Performance of the correlating receiver in a system with exponentially decreasing path strengths, a signal bandwidth of $\frac{10}{T}$, and the constant $\frac{S_0^2 a_0^2}{2N_0\tau} = 1$

4 Conclusions

4.1 Summary

This thesis has examined the performance of fiber optic networks with incoherent sources and multiple paths. The networks were modeled as having either a fixed number of equal strength paths or an infinite number of paths with exponentially decreasing strengths. The performance, defined as a signal-to-noise ratio, was determined as a function of the signaling interval, the differential delay, and the signal bandwidth. The performance was evaluated for three different receivers; the zero-forcing equalizer, the minimum-mean-square-error equalizer, and the correlating receiver.

The results show that the maximum performance for each receiver is directly proportional to the signal bandwidth. Furthermore, as the ratio of the differential delay to the signaling interval gets small, all three receivers approach their maximum performance. This is because as the differential delays gets small the amount of intersymbol interference is decreased. Also, as the signal bandwidth increases a smaller differential delay is needed to attain the maximum performance.

At a larger ratio of differential delay to signaling interval, the minimum-mean-square-error equalizer gives much better performance than the other two when there are equal strength paths. If the path strengths are exponentially decreasing, then the performance of the zero-forcing equalizer approaches that of the minimum-mean-square-error equalizer. Since the correlating receiver is not designed to decrease the effect of intersymbol interference, its performance is much worse than the other receivers in this case.

4.2 Future Research

Networks with multiple paths are currently being built in the LCN lab at MIT. These networks are being designed using topologies described by Wasem [4] and Ku [2]. Using these networks, it will be possible to verify experimentally the models used in this thesis and to determine whether networks of this type are feasible.

At the present time, there is a lot of research devoted to the development of lightwave communication systems that use coherent detection. Currently, this technology is not sufficiently developed to allow its use in networks with multiple paths. However, as coherent systems become further developed, these types of networks will become more interesting. Therefore, a natural extension of this research will include an analysis similar to that presented here for networks with coherent sources and coherent detection.

REFERENCES

[1] Robert S. Kennedy, Massachusetts Institute of Technology, Class notes for 6.450, 1987.

[2] May Ku, "Topology for Reliable Fiber-Optic Local Communication Network," Master's Thesis Proposal, Massachusetts Institute of Technology, Cambridge, December 1987.

[3] John G. Proakis, *Digital Communications*, McGraw-Hill Book Company, U.S.A., 1983.

[4] Ondria J. Wasem, "Topologies for Fiber Optic Local Communication Networks with no Internal Switching," Ph.D. Thesis Proposal, Massachusetts Institute of Technology, Cambridge, September, 1987.

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